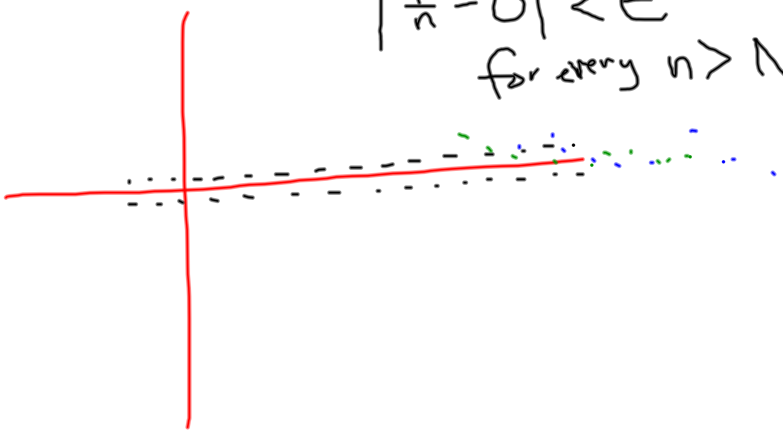
to show the $\lim_{n\to\infty} \frac{1}{n} = 0$

then you must show for every $\epsilon > 0$ greek letter epsilon

there is an N such that

$$\left|\frac{1}{n} - 0\right| < \epsilon$$

for every $n > N$.



to show the $\lim_{n\to\infty} \frac{1}{n} = 0$

then you must show for every $\epsilon > 0$ (greek letter epsilon)

there is an N such that

$\left|\frac{1}{n} - 0\right| < \epsilon$

for every $n > N$.

suppose we have an $\epsilon > 0$

$-\epsilon < \frac{1}{n} < \epsilon$

$\left|\frac{1}{n} - 0\right| < \epsilon$

$\left|\frac{1}{n}\right| < \epsilon$

$\frac{1}{n} < \epsilon$

$N_{\text{int}} > \frac{1}{\epsilon} < n$

l'Hospital

l'Hôpital

Sequence

f^n where the domain is only integers.

ordered list of numbers.

$$\left\{\frac{1}{n}\right\}_{n=1}^{\infty} = 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}, \ldots$$

$$a(n) = \{a_n\}_{n=1}^{\infty} = a_1, a_2, a_3, \ldots$$

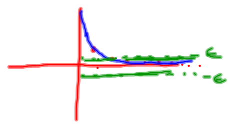
$$\{a_n\}_{n=1}^{\infty} = \left\{\frac{1}{n^2}\right\}_{n=1}^{\infty}$$

Convergence

A sequence $\{a_n\}$ converges to L,

if $\lim_{n\to\infty} a_n = L$

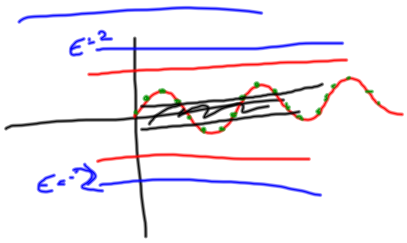
graph $\left\{\frac{1}{n}\right\}$



if $\lim_{x\to\infty} a(x) = L$

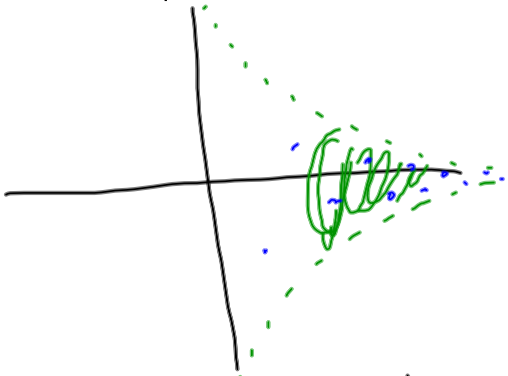
then $\lim_{n\to\infty} a_n = L$

$[\text{when } a_n = a(n)]$



A sequence can diverge

* when $\lim = \pm\infty$
* oscillation



if $\lim |a_n| = 0$

then $\lim a_n = 0$

$$\left\{ \frac{(-1)^n}{n} \right\}_{n=1}^{\infty}$$

$-1, \frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, \ldots$

$n! = n \cdot (n-1) \cdot (n-2) \cdots 1$

$2! = 2 \cdot 1 = 2$

$3! = 3 \cdot 2 \cdot 1 = 6$

$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

$\vdots$

by convention,

$0! = 1$

$1! = 1$

Ex. 1.9 in 8.1

show that $\left\{ \frac{n!}{n^n} \right\}_{n=1}^{\infty}$ converges

$\frac{1!}{1^1}, \frac{2!}{2^2}, \frac{3!}{3^3}, \ldots$

$1, \frac{1}{2}, \frac{2}{9}, \ldots$

$$\frac{n!}{n^n} = \frac{n \cdot (n-1)(n-2) \cdots (1)}{n \cdot n \cdot n \cdot n \cdots n}$$

$$\lim_{n\to\infty} \frac{n!}{n^n} = \lim_{n\to\infty} \frac{n(n-1) \cdots (1)}{n \cdot n \cdot n \cdot n} < \lim_{n\to\infty} (1)(1)(1) \cdots \left(\frac{1}{n}\right)$$

$$= \lim_{n\to\infty} \frac{1}{n} = 0$$

we know ... defn of sequences

defn of sequence convergence

"Proving that a sequence converges" (optional)

$$\lim_{x\to\infty} f(x) = L \Rightarrow \lim_{n\to\infty} f(n) = L$$

L'Hospital's Rule

Squeeze Theorem

$$\lim_{n\to\infty} |a_n| = 0 \Rightarrow \lim_{n\to\infty} a_n = 0$$

$$n! = n(n-1)(n-2)\cdots(1)$$

☆ $0! = 1$ by definition

A sequence $\{a_n\}_{n=1}^{\infty}$ increases if

$$a_1 \le a_2 \le a_3 \le \cdots \le a_n \le a_{n+1} \le \cdots$$

A sequence $\{a_n\}_{n=1}^{\infty}$ decreases if

$$a_1 \ge a_2 \ge a_3 \ge \cdots \ge a_n \ge a_{n+1} \ge \cdots$$

(strictly increasing/decreasing) if no $=$)

A sequence is MONOTONIC if it is either inc. or dec.

Bounded

A seq $\{a_n\}$ is BOUNDED if there is an $M > 0$ and $|a_n| \le M$ for every n

$\{\sin(n)\}$ has a bound of: 1, 2, e, π, 10,000

Completeness: every sequence bounded above has a LEAST UPPER BOUND

every seq... below has a GREATEST LOWER BOUND

EVERY BOUNDED MONOTONIC sequence CONVERGES

Infinite Series - sequences

2011-01-24 Pd 2